

The RMS Charge Radius of the Proton and Zemach Moments

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Abstract

On the basis of recent precise measurements of the electric form factor of the proton, the Zemach moments, needed as input parameters for the determination of the proton rms radius from the measurement of the Lamb shift in muonic hydrogen, are calculated. It turns out that the new moments give an uncertainty as large as the presently stated error of the recent Lamb shift measurement of Pohl *et al.* De Rújula's idea of a large Zemach moment in order to reconcile the five standard deviation discrepancy between the muonic Lamb shift determination and the result of electronic experiments is shown to be in clear contradiction with experiment. Alternative explanations are touched upon.

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1 Introduction

Recently, two new precise measurements of the proton root mean square (rms) radius have been published which deviate by 5 standard deviations. The first one studying muonic hydrogen resulted after a ten years effort in $r_p = \sqrt{\langle r^2 \rangle} = 0.84184(67)$ fm [1], a factor of about 10 more precise than all previous determinations. These previous determinations are based on three very different methods: The CODATA values are derived mostly from electronic hydrogen giving $r_p = 0.8768(69)$ fm [2], the Lamb shift of electronic hydrogen resulting in $r_p = 0.883(14)$ fm [3, 4], and electron scattering from hydrogen yielding $r_p = 0.895(18)$ fm [5]. These “electronic” determinations were recently corroborated by a new precise determination from electron scattering at the Mainz Microtron MAMI giving $r_p = 0.879(8)$ fm [6]. Even considering the larger error of the electronic determinations the deviation of ≈ 0.04 fm has a significance of five standard deviations. This unexpected result created quite a stir in the hadron community and asks for explanation.

The following proposals have been ventilated [7]:

- *The experimental results are not right.*
This is highly unlikely. The electronic experiments are based on well established methods and the agreement of their results cannot be accidental. The muonic experiment represents a very sophisticated study of a group beyond any doubts.
However, in a followup paper to ref. [7] De Rújula [8] questions just the correctness of the electron-proton scattering (ep) results and a refutation of his conjectures is one of the aims of this paper.

- *The relevant QED calculations are incorrect.*

It is true that all determinations with electromagnetic probes require higher order corrections, mostly called “radiative corrections”. However, in the case of the electronic determinations the methods used are completely different requiring as different corrections. This is particularly true for the about half dozen electron scattering experiments performed in different laboratories at very different kinematic conditions. The agreement of the electronic experiments cannot be by chance.

The QED calculations, on which the Lamb shift determination in muonic hydrogen is based, are very detailed and sophisticated and have been improved again and again over more than four decades. However, the perturbative expansion requires the knowledge of higher order radial moments of the charge distribution, so called Zemach moments [9, 10]. In a recent paper De Rújula argued that the solution of the problem might be found in a third Zemach moment as large as $\langle r^3 \rangle_{(2)} = 36.6 \text{ fm}^3$ [7]. Immediately after that publication Cloët and Miller [11] showed that the proposal of De Rújula is in contradiction with experiment. Nevertheless, De Rújula insisted on his arguments by suggesting that the experimental basis is wrong [12, 8]. We shall show on the basis of the most recent, precise experimental results of Bernauer *et al.* [6] that this is not true and this is the central subject of this paper.

It is noteworthy that QED considerations show [13, 14] that the electron scattering experiments and the atomic hydrogen spectroscopy determine the same radius.

- *There is, at extremely low energies and at the level of accuracy of the lepton-proton atom experiments, “physics beyond the standard model”.*

After having excluded the previous point there stays the need for an explanation which we will briefly discuss in the conclusions. However, it is the conviction of the present authors that one has to investigate more ideas within the standard model before one may venture in physics beyond it.

- *A single-dipole form factor is not adequate to the analysis of precise low-energy data.*

This is definitely true as we shall show in this paper. However, the small deviations from the standard one-dipole form [6] will change the input into the determination of the muonic Lamb shift only very mildly, but are at dramatic variance with the proposal of De Rújula [7].

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2 Summary of our knowledge of the electric form factor of the proton

Before we can turn to the central point, we have to summarise our knowledge about the electric form factor of the proton $G_{E,p}$. This knowledge started with the key paper of Janssens *et al.* [15] using the Rosenbluth separation of the electric and magnetic form factors in electron scattering. They found as the shape of $G_{E,p}$ what is called today the standard dipole form $G_{E,p} = 1/[1 + Q^2/(0.71 \text{ (GeV/c)}^2)]^2$. After quite a few further determinations this shape stayed the accepted reference until about the year 2000. It was a great surprise that this reference had to be modified after experiments performed at Jlab using polarised electrons with polarised targets or by measuring the polarisation transfer to the recoil proton. They showed a dramatic faster decline than the standard dipole form at large Q^2 . Several reviews have been published on this topic in the last decade [16, 17, 18] giving access to the extended literature.

At low momentum transfer squared Q^2 a particularly precise determination could be reached recently by fitting a large variation of form factor models to the measured differential cross sections directly, i.e. extending the Rosenbluth method [6]. This determination showed some structure at scales of the pion cloud and it is this scale which is considered to be particularly essential for the correct determination of the Zemach moments. However, as we will show, the form factor shape is important at all scales due to the features of the Fourier transform. It is needed for the derivation of the charge distribution, indispensable for the calculation of the Zemach moments from the measured form factors. It turns out that the low Q^2 shape influences the precision of the Zemach moments, but the bulk behaviour is essentially insensitive to the specific model chosen.

For large Q^2 we use the results based on the fit of the pre-2007 data by Arrington *et al.* [18], however, without two photon corrections. The possible influence of these still unsafe corrections is small in the relevant Q^2 range and does not change the results in this paper.

Figure 1 shows the standard dipole form and the shape used in this paper. This shape consists of the Bernauer *et al.* results for $Q < 0.7 \text{ GeV/c}$ and of the fits of Arrington *et al.* for $Q \geq 0.7 \text{ GeV/c}$. From the many models of Bernauer *et al.* which are equivalent within the statistics, we have taken the standard dipole times a power series of order 8 to produce the plots and the numerical results. If errors are quoted for the Zemach moments given in section 3 the full spread of models is considered with and without readjustment of the Arrington *et al.* normalisations by as much as 2%. In the future, a new global fit including all data is highly desirable, however, as checks show, for the purpose of this letter no better information is needed.

The shapes of Bernauer *et al.* and Arrington *et al.* differ a few per mill at small Q as shown in the linear plot in Fig. 2.

With these form factor parametrisations we can now determine the Zemach moments needed for the calculation of the hyperfine splitting and the Lamb shift in hydrogen [9, 10].

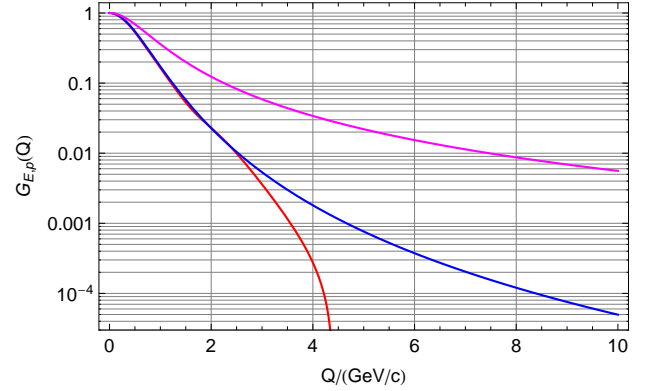


Figure 1: The electric form factor of the proton $G_{E,p}$ as given by the standard dipole form (blue curve) and as given by the assembly of the fits of Bernauer *et al.* and Arrington *et al.* (red curve). The toy model of De Rújula (magenta curve) will be dealt with in section 4. For details see text.

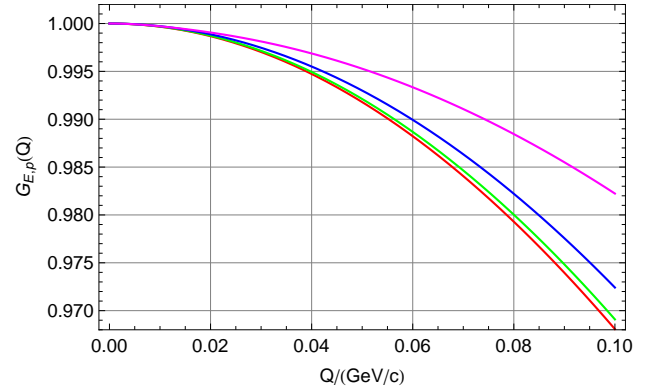


Figure 2: As Fig. 1 but in linear scale for small Q . The red curve depicts the Bernauer *et al.* form and the green that of Arrington *et al.*. The blue curve is again the standard dipole and the magenta curve the toy model of De Rújula.

3 Zemach moments

The Zemach moments of the nuclear charge distribution come about through the smearing of the Coulomb potential with the extended charge distribution and the perturbative expansion of the hydrogen wave functions [9, 10]. They are defined by

$$\langle r^n \rangle_{(2)} = \int d^3r r^n \rho_{(2)}(r) \quad (1)$$

where $\rho_{(2)}(r)$ is the convolution of the charge or magnetic distribution for the Lamb shift or hyperfine interaction, respectively:

$$\rho_{(2)}(r) = \int d^3r_2 \rho_{\text{charge}}(|\vec{r} - \vec{r}_2|) \rho_{\text{charge or magnetic}}(r_2). \quad (2)$$

Since we are at this point concerned with the Lamb shift, the proton charge distribution is folded with itself for the

calculation of $\rho_{(2)}(r)$. Inserting the Fourier transform of the Sachs electric form factor $G_E(Q^2)$ for ρ in Eq. (2), and integrating repeatedly by parts, one finds that the first and the third Zemach moment can also be expressed in momentum space [20]:

$$\langle r^1 \rangle_{(2)} = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} (G_E^2(Q^2) - 1) \quad (3)$$

$$\langle r^3 \rangle_{(2)} = \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^4} \left(G_E^2(Q^2) - 1 + \frac{Q^2}{3} \langle r^2 \rangle \right). \quad (4)$$

These integrals can be evaluated analytically for the dipole form and the monopole form (see section 4). For the general form factors of section 2 we integrate Eqs. (3) and (4) numerically.

For illustration, we show in Fig. 3 the charge distribution $\rho_{\text{charge}}(r)$ and the folded distribution $\rho_{(2)}(r)$ for the assembly of the $G_{E,p}$ fits of Bernauer *et al.* and Arrington *et al.* depicted in Fig. 1.

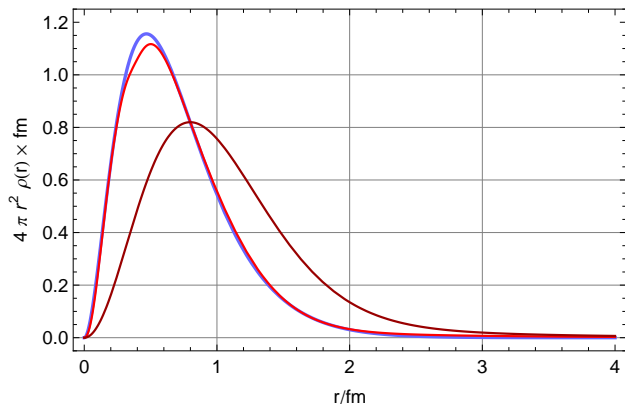


Figure 3: The charge distribution $\rho_{\text{charge}}(r)$ (red curve), the folded distribution (dark red curve) $\rho_{(2)}(r)$ for the assembly of the $G_{E,p}$ fits of Bernauer *et al.* and Arrington *et al.* (red curve in Fig. 1), and the standard dipole distribution (blue curve) are shown. The stability of the numerical method used to perform the Fourier transform can be seen from the flatness of the curve at larger radii. The shift of charge from low radii to the tail (of the light red curve) can be interpreted as “pion cloud”, however, such an interpretation needs a careful qualification [19].

It is instructive to study the integrand of Eq. (4) and learn where $G_E^2(Q^2)$ contributes. The third Zemach moment calculated from the assembly of Bernauer *et al.* and Arrington *et al.* fits is a factor 1.4 larger than the standard dipole reference. For this form about 60% of the integral arise from momentum transfers $Q < 0.7 \text{ GeV}/c$, but 75% of the surplus originate in this interval and 50% from $Q < 0.3 \text{ GeV}/c$. This demonstrates the particular importance of low Q electron scattering data for the evaluation of the Zemach moments, an observation already rightly emphasized by De Rújula in Ref. [12].

By numerical integration of the form factors according to Eqs. (3) and (4) and the folded distribution in Eq. (1) we get the moments listed in Table 1. A good cross check of the numerical accuracy is the fulfillment of the identity $\langle r^2 \rangle_{(2)} = 2 \langle r^2 \rangle$ [9]. For the moments in Table 2 the weighted charge distribution derived by numerical Fourier transform of the Arrington-Bernauer parametrisation is integrated numerically. Where possible the calculations were compared to evaluations in momentum space, e.g. $\langle r^2 \rangle$ can also be determined from the slope of G_E at $Q^2 = 0$. While the agreement is very good for the moments of lower order, we find that for higher orders the dependence on the form factor models gets more important and the numerical stability gets worse resulting in larger errors. For $\langle r^5 \rangle$ we estimate a numerical uncertainty comparable to the statistical error and for $\langle r^6 \rangle$ this uncertainty is as big as 1.7 times the statistical error.

The rms radius is derived from the measured Lamb shift via the numerical equation [22]

$$L^{5th}[\langle r^2 \rangle, \langle r^3 \rangle_{(2)}] = \left(209.9779(49) - 5.2262 \frac{\langle r^2 \rangle}{\text{fm}^2} + 0.00913 \frac{\langle r^3 \rangle_{(2)}}{\text{fm}^3} \right) \text{meV}. \quad (5)$$

showing that in particular the moments $\langle r^2 \rangle$ and $\langle r^3 \rangle_{(2)}$ are of interest here. For the Zemach moment $\langle r^3 \rangle_{(2)}$, unknown without the knowledge of the charge distribution, the approximation $\langle r^3 \rangle_{(2)} \approx f \langle r^2 \rangle^{3/2}$ has been adopted in Ref. [1]. The factor f used in the extraction of the muonic rms radius was $f = 3.79$, i.e. in accord with that of the standard dipole and that of Friar and Sick (see Table 1). The new precise experiment of Bernauer *et al.* yields an improved value of $f = 4.18(13)$. This new value would increase the r_p in Ref. [1] by $+0.00025 \text{ fm}$ – only a minor correction within the margin of the quoted theoretical error. One could be tempted to use the new experimental value for the third Zemach moment $\langle r^3 \rangle_{(2)}$ and “improve” the muonic result to give $r_p = 0.84245(67) \text{ fm}$, but this would be somewhat inconsistent as we insist on the larger radius and the corresponding form factor obtained from electron scattering. If one would interpret this difference as a lack of knowledge of $\langle r^3 \rangle_{(2)}$ one had to add it as a systematic error in linear and double the error in Ref. [1].

Although not essential for the purpose of this paper, but of general interest in atomic physics, we have calculated additional moments of the proton electromagnetic distributions. The moments listed in Table 2 are required for calculating the higher order finite size corrections to the Lamb shift and to the hyperfine splitting in electronic and muonic hydrogen. Some of the values differ significantly from the standard dipole approximation.

4 De Rújula’s toy model

We now turn to the “toy” model of De Rújula [7] designed to explain the difference between the electronic results and the muonic determination.

The basic, very physical idea is the assumption of a charge cloud reaching out to very large scales of $\approx 20 \text{ fm}$. De Rújula designs the charge distribution in his toy model

Table 1: Moments of the charge distribution and Zemach moments of the respective parametrizations of the form factor $G_{E,p}$. All distribution functions are duly normalising to one. The ratio f has been used in Ref. [1] for the approximation of the third Zemach moment. Errors for the Bernauer-Arrington fit assembly have been determined using the full spread of the form factor fits in Ref. [6]. The moments for De Rújula’s toy model were calculated with parameters typical for the CODATA rms radius: $M = 0.750 \text{ GeV}/c^2$, $m = 0.020 \text{ GeV}/c^2$, and $\sin^2(\theta) = 0.3$.

$G_{E,p}$	$\langle r^2 \rangle / \text{fm}^2$	$\langle r^1 \rangle_{(2)} / \text{fm}$	$\langle r^2 \rangle_{(2)} / \text{fm}^2$	$\langle r^3 \rangle_{(2)} / \text{fm}^3$	$f = \langle r^3 \rangle_{(2)} / \langle r^2 \rangle^{3/2}$
Standard dipole [15]	0.6581	1.0246	1.316	2.023	3.789
Friar-Sick [21]	0.801(36)			2.71(13)	3.78(31)
Arrington [18]	0.742	1.077	1.484	2.50	3.91
Bernauer-Arrington [6, 18]	0.774(8)	1.085(3)	1.553(16)	2.85(8)	4.18(13)
Toy model	0.771	0.807	1.542	36.2	53.5
De Rújula [7]	0.7687			36.6(7.7)	52.2

in such a way that the rms radius is constrained by the rms radii of Sick [5] or that of CODATA [2]. The third Zemach moment is made so large, i.e. $\langle r^3 \rangle = 36.6 \text{ fm}^3$, that the rms radius derived from the muonic Lamb shift (see Eq. (5)) would be in accord with the electronic results. This increase is accomplished by adding to a “Yukawian charge distribution”, corresponding to a monopole form factor, with short range, an exponential distribution, corresponding to a dipole form factor, with long range. As illustration Fig. 4 shows one typical De Rújula charge distribution compared to the distribution derived by Fourier transform from the measured $G_{E,p}$ presented in section 2.

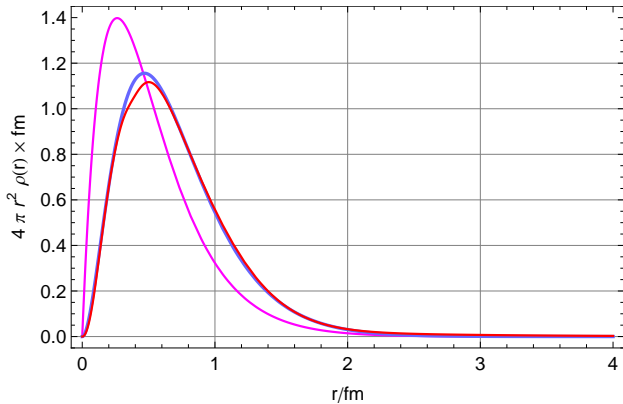


Figure 4: The charge distribution derived from the Fourier transform of the Bernauer-Arrington parametrisation of $G_{E,p}$ (red curve) and the charge distribution of the toy model of De Rújula [7] (magenta curve). The De Rújula charge distribution was calculated with parameters given in the caption of Table 1. For comparison the charge distribution of the standard dipole form factor is shown (blue curve).

The two charge distributions are in dramatic disagreement. In order to make the disaccord more visible for the experimentally observable $G_{E,p}$ we show in Fig. 1 the Fourier transform of the De Rújula charge distribution compared to the $G_{E,p}$ of the Bernauer-Arrington parametrisation and the

standard dipole form. It is evident that the De Rújula model is excluded by experiment. The same was already shown on the basis of older data and fits by Cloët and Miller [11].

The basic problem of the De Rújula argumentation is the design of his model by the constraint of two range momenta only, instead of using the full information available in the form factors. The scale arguments are easily leading astray since scales define wave trains. One needs the full momentum spectrum in a Fourier transform to describe an object of finite extent. In his most recent followup paper [8] De Rújula concluded with the request *... it would be very helpful to extract the correlation dictated by the ep data ...* and this is exactly provided in this paper, however, in the complete way of form factors going much beyond the correlations of just two moments.

Not being able to maintain his large third Zemach moment on the basis of experiment he takes the attitude that the experiments are wrong or at least not sufficiently precise to exclude his conjectures [8]. Of the several inaccuracies in his arguments we have to rectify three:

- The *dark trumpet* in Fig. 1 of Bernauer *et al.* [6] includes all errors and represents new, precise ep cross section data. The extrapolation to $Q^2 = 0$ determining $\langle r^2 \rangle$ is much less arbitrary than insinuated in Fig. 2 of Ref. [8] since the red arrows neglect the normalisation $G_E(Q^2 = 0) = 1$ imposed by the total charge of the proton. Since no experiment could determine the cross sections with an absolute precision better than a few percent, they are normalised to $G_E(Q^2 = 0) = 1$ using a certain analytical hypothesis for the form factor shape. As for the determination of the rms radius from the small Q^2 behaviour this has been amply discussed in Ref. [5].
- The p-value of the χ^2 distribution is utilised as an important argument at several places in the paper, however, its meaning is not correctly applied. In short: A $\chi^2 \gg n_{dof}$ means either that the fit model is insufficient and/or that the data sample has larger than statistical errors. Almost all experiments have systematic errors and by increasing the errors in Ref. [6] by as little as 7% one would get a $\chi^2/n_{dof} = 1$ indicating a small systematic error indeed.

The issue concerns the difference between the frequentist and the Bayesian interpretation of a fit. Additionally, in hypothesis testing the p-value does not give the probability that the null hypothesis is true. A good reference is the CERN classic of Frederick James [23].

- The rms radius of Ref. [6] includes the correct and complete errors of a detailed error analysis compatible with the book mentioned above.

Table 2: Various moments of the charge distribution for the standard dipole form and the assembly of the fits of Bernauer *et al.* and Arrington *et al.*. All units are in powers of fm. Note that $\langle r \rangle_{(2),em}$ represents the so called Zemach radius calculated with the corresponding Eq. (3) for the electric and magnetic form factors. An additional numerical uncertainty is given for the highest moments.

	standard dipole	Bernauer- Arrington
$\langle 1/r^2 \rangle$	9.117	8.100(19)
$\langle 1/r \rangle$	2.135	2.0666(24)
$\langle r \rangle$	0.7026	0.7381(17)
$\langle r^2 \rangle$	0.6581	0.774(8)
$\langle r^3 \rangle$	0.7706	1.16(4)
$\langle r^4 \rangle$	1.083	2.59(19)(04)
$\langle r^5 \rangle$	1.775	8.0(1.2)(1.0)
$\langle r^6 \rangle$	3.325	29.8(7.6)(12.6)
$\langle \log(r) \rangle$	-0.5289	-0.4944(12)
$\langle r^2 \log(r) \rangle$	0.0358	0.169(12)
$\langle r \rangle_{(2),em}$	1.025	1.045(4)

5 Conclusion

Having disproven the proposal of De Rújula by the experiments again, the question arises where else one could find a reconciliation of the two disagreeing radii. One question is, of course, whether the QED corrections are complete or whether some higher order contributions are not yet sufficiently well studied. The most recent, comprehensive discussion of these possible deficiencies can be found in the paper by Edith Borie [22]. One might add that it could be necessary that the distorted relativistic wave functions had to be treated in a perturbative sum over multiphoton exchanges. The convergence of such sums is not proven in QED. Such a calculation may imply the solution of a Bethe-Salpeter equation and appears hardly possible [24]. However, further theoretical studies may finally show that the approximations made so far are adequate.

In this case, one had to go to fancier ideas but still within the framework of the standard model. We mention the so called “hadronic corrections” as the polarisation of the proton or mesonic loops. We could, for example, assume a further particle-antiparticle fluctuation in the point Coulomb field, i.e. beyond the e^+e^- and $\mu^+\mu^-$

pairs included in the calculations of the Lamb shift so far. However, inserting the π mass, the lightest hadron known, in the nonrelativistic approximation of Pachucki [25] gives a small contribution of -0.0095 meV only, already taken into account in Eq. (5) [24]. If we assign the difference of the two determinations of the radius of 0.038 fm fully to the energy difference of the $2S - 2P$ point nucleus Lamb shift we get a shift of -0.341 meV. The mass of an electrically charged particle-antiparticle pair producing such a Lamb shift would be 23 MeV. No free particle with this mass is known.

However, quantum mechanics demands fluctuations of quarks-antiquarks also in the Coulomb field and one might identify these particles with quarks. It is not evident that these pairs have to form the asymptotic Goldstone Boson, the pion, resulting from the breaking of chiral symmetry of QCD, if this is produced dynamically, i.e. including gluonic components. One could think of the vacuum in terms of a Fock state expansion and consider the contribution to the vacuum polarisation as a sum of terms yielding a low effective mass. Inserting the quark charges and assuming $2m_{up} = m_{down}$ one gets $m_{up, effective} = 28 \text{ MeV}/c^2$. Considering the small energy scales in muonic hydrogen this does not unreasonably compare to the current quark masses $m_{up} \approx 3 \text{ MeV}/c^2$ and $m_{down} \approx 6 \text{ MeV}/c^2$ at the 2 GeV scale [26]. If such QCD loops are present one has to investigate their influence on the QED corrections for the hyperfine splitting of muonic hydrogen as well.

Ironically, one could revert the interpretation of the muonic hydrogen experiment if we assume that the QED calculations are sufficiently exact. By inserting the precise radius from the electronic experiments and the safe Zemach moments presented in this paper, one can determine the polarisation in the Coulomb field by quark loops or other hadronic, possibly dual, corrections at a very low Q^2 scale. This idea needs a thorough theoretical study going beyond the scope of this paper.

Acknowledgement

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